Here the number of terms is $\frac{a}{2x} + 1$, and the sum

$= \frac{1}{2}\left(\frac{a}{2x} + 1\right)\left(\frac{a}{2x} + 2\right) =$ number of positions in which the distance between the points is *not less* than half the line.

But the whole number of positions as seen before is $\frac{a}{2x^2}\ (a + x)$.

When x is taken very small, these quantities reduce to $\frac{a^2}{8x^2}$ and $\frac{a^2}{2x^2}$, the first part of which is $\frac{1}{4}$ the second, hence the whole number of positions in which the greatest distance is *less* than half the line is $\frac{3}{4}$ of the whole number of possible positions.

SOLUTIONS OF PROBLEMS IN NO. 7, AND 29, IN NO. 6.

Solutions of problems in No. 7 have been received as follows:

From Geo. L. Dake, 30, 31 & 32; Geo. M. Day, 30, 32 & 33; Prof. A. B. Evans, 30, 31, 32 & 33; Prof. H. T. Eddy, 32; Henry Gunder, 30, 31, 32 & 33; H. Heaton, 30, 31, 32 & 33; Prof. E. W. Hyde, 32 & 33; Prof. D. Kirkwood, 31; Artemas Martin, 30, 31, 32 & 33; L. Regan, 30 & 31; Walter Siverly, 30, 31, 32 & 33; Werner Stille, 30, 31, 32 & 33; E. B. Seitz, 30. 31, 32 & 33; Prof. D. Trowbridge, 30, 31 & 32.

Elegant solutions of all the questions proposed in No. 6 were received in due time, last month, from Walter Siverly, but the letter containing his solutions of 25, 26 and 29 was mis-laid and hence proper credit was not given him in No. 8. Mr. Martin also sent correct solutions of Nos. 25 and 26, but failed to get credit for them in No. 8, because his letter, also, was mis-laid.

29. "If $a, b, c, d, e, f, g, h, i, j, k$ be chords drawn from any point on the circumference of a circle to the eleven angles of an inscribed regular polygon of eleven sides; prove that

$$(a + k)(b + j)(c + i)(d + h)(e + g) = f^5 \dots\dots\dots\dots\dots\dots (1)."$$

GENERAL SOLUTION, BY PROF. A. HALL.

Put the radius of the circle equal to unity, and let the number of the sides of the polygon be n, an odd number. Then $\cos\theta$ and $\sin\theta$ are the coordinates of a point on the circumference, and

$$(1,0);\left(\cos\frac{2\pi}{n},\ \sin\frac{2\pi}{n}\right);\ \left(\cos\frac{4\pi}{n},\ \sin\frac{4\pi}{n}\right)\ldots\left(\cos\frac{(n-1)\pi}{n},\ \sin\frac{(n-1)\pi}{n}\right),$$

are the coordinates of the corners of the polygon. Denote the chords by c_1; c_2; c_3;........c_n. We shall have by the common formulae of analytical geometry, or from the geometry of the figure,

$$c_1 = 2\sin\frac{1}{2}\theta, \qquad c_2 = 2\sin\left(\frac{\theta}{2}-\frac{\pi}{n}\right),$$

$$c_3 = 2\sin\left(\frac{\theta}{2}-\frac{2\pi}{n}\right), \qquad c_4 = 2\sin\left(\frac{\theta}{2}-\frac{3\pi}{n}\right),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$c_{n-1} = 2\sin\left(\frac{\theta}{2}-\frac{(n-2)\pi}{n}\right), \qquad c_n = 2\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{n}\right).$$

Adding these values in the manner indicated we have

$$c_1 + c_n = 4\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{2n}\right)\cos\frac{(n-1)\pi}{2n},$$

$$c_2 + c_{n-1} = 4\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{2n}\right)\cos\frac{(n-3)\pi}{2n},$$

$$c_3 + c_{n-2} = 4\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{2n}\right)\cos\frac{(n-5)\pi}{2n},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$c_{\frac{n-1}{2}} + c_{\frac{n+3}{2}} = 4\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{2n}\right)\cos\frac{2\pi}{2n},$$

and for the middle chord,

$$c_{\frac{n-1}{2}} = 2\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{2n}\right).$$

The product of the sums is therefore

$$2^{n-1}.\sin\left(\frac{\theta}{2}-\frac{(n-1)\pi}{2n}\right)^{\frac{n-1}{2}}.\cos\frac{(n-1)\pi}{2n}.\cos\frac{(n-3)\pi}{2n}.\cos\frac{(n-5)\pi}{2n}\ldots\cos\frac{2\pi}{2n}$$

Now from a known theorem we can deduce easily the equation,

$$2^{\frac{n-1}{2}} \cdot \cos\frac{(n-1)\pi}{2n} \cdot \cos\frac{(n-3)\pi}{2n} \cdot \cos\frac{(n-5)\pi}{2n} \cdots\cdots \cos\frac{2\pi}{2n} = 1,$$

and therefore we have

$$c^{\frac{n-1}{2}}_{\frac{n+1}{2}} = (c_1 + c_n)(c_2 + c_{n-1})(c_3 + c_{n-2})(c_4 + c_{n-3}) \ldots (c_{\frac{n-1}{2}} + c_{\frac{n+3}{2}}).$$

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30. "Prove that

$$\frac{\sqrt[4]{a} + \sqrt[4]{b}}{\sqrt[4]{a} - \sqrt[4]{b}} = \frac{a + b + 2\sqrt{ab} + 2\sqrt[4]{a^3 b} + 2\sqrt[4]{ab^3}}{a - b}."$$

SOLUTION BY GEO. L. DAKE, CLEVELAND, OHIO.

Multiply both numerator and denominator of the given fraction by $(\sqrt{a} + \sqrt{b})(\sqrt[4]{a} + \sqrt[4]{b})$, reduce, and we shall have

$$\frac{a + b + 2\sqrt{ab} + 2\sqrt[4]{a^3b} + 2\sqrt[4]{ab^3}}{a - b}.$$

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31. "Solve the equation

$$x = \sqrt[4]{x + \cfrac{x}{\sqrt[4]{x + \cfrac{x}{\sqrt[4]{x + \&c}}}}}$$

and express the value of x in a finite number of terms."

SOLUTION BY ASHER B. EVANS, LOCKPORT, N. Y.

By the theory of continued fractions the given equation becomes

$$x = \sqrt[4]{x + x \div x} = \sqrt[4]{x+1} \quad \therefore x^4 = x + 1 \ldots\ldots\ldots\ldots (1).$$

Assume $$(x^2 + p)^2 = (qx + r)^2 \ldots\ldots\ldots\ldots\ldots (2).$$

Equations (1) and (2) will be identical when $2p = q^2$, $2qr = 1$, and $r^2 - p^2 = 1$. By eliminating q and r these three conditions give

$$8p^3 + 8p - 1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

Put $2p = y - \frac{4}{3y}$; then (3) becomes $y^6 - y^3 = \frac{64}{27}$; whence

$y = \left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{283}{27}}\right)^{\frac{1}{3}}$; and therefore

$$p = \frac{1}{2}y - \frac{2}{3y} = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{283}{27}}\right)^{\frac{1}{3}} + \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\sqrt{\frac{283}{27}}\right)^{\frac{1}{3}} \ldots\ldots(4).$$

Since $q = \sqrt{2p}$ and $r = \frac{1}{2q} = \frac{1}{2\sqrt{2p}}$, we find from (2) by substituting these values of q and r

$$(x^2 - p)^2 = \left(x\sqrt{2p} + \frac{1}{2\sqrt{2p}}\right)^2.$$

$$\therefore x^2 - x\sqrt{2p} = \frac{1}{2\sqrt{2p}} - p \text{ and}$$

$$x^2 + x\sqrt{2p} = -\frac{1}{2\sqrt{2p}} - p; \text{ whence}$$

$$x = \sqrt{\frac{1}{2}}\left(\sqrt{p} \pm \sqrt{\frac{1}{\sqrt{2p}} - p}\right) \text{ and}$$

$$x = \sqrt{\frac{1}{2}}\left(-\sqrt{p} \pm \sqrt{-\frac{1}{\sqrt{2p}} - p}.\right.$$

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32. "There are two spheres of equal size and of exactly the same appearance, one solid silver and galvanized with gold and the other hollow and made of gold. Required some means by which one may be determined from the other."

SOLUTION BY PROF. D. TROWBRIDGE, WATERBURGH, N. Y.

Convert the spheres into pendulums, and let a be the distance from the point of suspension to the centre of the spheres, and l the distance from the same point to the centre of oscillation in the solid sphere and l' in the hollow one, k the principal radius of gyration for the solid sphere, and k' for the other. Then

$$l = \frac{k^2 + a^2}{a}, \qquad l' = \frac{k'^2 + a^2}{a}.$$

If r and r' be the radii of the external and the internal surface of the hollow sphere, then

$$k^2 = \frac{2}{5}r^2, \text{ and } k'^2 = \frac{2}{5}\cdot\frac{r^5 - r'^5}{r^3 - r'^3} = \frac{2}{5}\left(r^2 + r'^2) - \frac{r^2 r'^2}{r^2 + rr' + r'^2}\right).$$

$\therefore$ $k' > k$, and $l' > l$; or the solid sphere will vibrate in the less time.

[H. T. Eddy, Prof. of Math. and Astronomy, Cincinnati, Ohio, has favored us with an elaborate solution of this question. Prof. Eddy supposes the spheres to be rolled down an inclined plane and shows that the solid ball will roll more rapidly than the other and reach the bottom more quickly. He obtains the eq., $t' = \sqrt{(k'^2 + r^2) \div (k^2 + r^2)}\,t$, in which t is the time and k the radius of gyration of the silver sphere, and t' the time and k' the radius of gyration of the golden sphere; and, assuming the spheres to be of aqual weight, he finds $k^2 = \frac{2}{5}r^2$ and $k'^2 = \frac{2}{5}\frac{781}{592}r^2$, and hence $t' = 1.044t$, nearly. We intended to publish this solution in full, but find it too extended for our space.—ED.]

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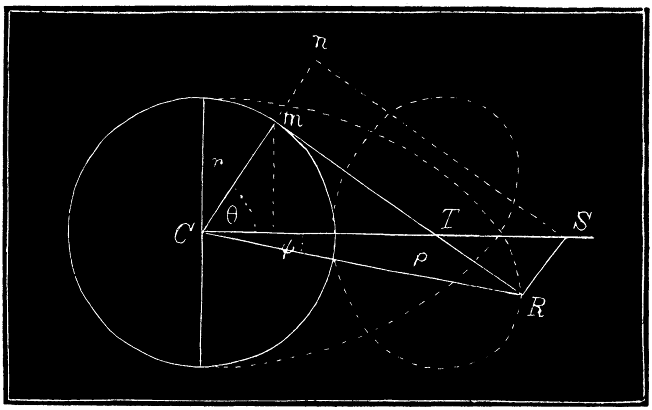
33. "A circle is referred to rectangular axes passing through the center. A tangent and ordinate are drawn from any point of the circumference. A distance n times the abscissa of this point is measured upon the axis of x and from this point a perpendicular falls upon the tangent. Required the equation of the locus of the intersection of the perpendicular and tangent."

SOLUTION BY WERNER STILLE, MARINE, ILL.



Let MTR be the tangent at M; CS $= nx$; MCS$=\theta$; SCR $=\psi$; then from the triangle CMR, when CM $= r$, and CR $= \rho$,

(1)............$\rho.\cos(\psi + \theta) = r.$

Also, when NS = MR, because $x = r\cos\theta$, CS $= n.r.\cos\theta$.

(2)............$\rho.\sin(\psi + \theta) = n.r.\cos\theta.\sin\theta.$

(2) can be written in the form, $\rho.\sin(\psi + \theta) = \frac{1}{2}\, n.r.\sin 2\theta.$

From this we easily deduce

$$\cos(\psi + \theta) = \sqrt{1 - \frac{n^2r^2}{4\rho^2}.\sin^2 2\theta},$$

which, when inserted in (1), gives

$$\rho = \frac{r}{\sqrt{1 - \frac{n^2r^2}{4\rho^2}.\sin^2 2\theta}}.$$

Finally, resolving to a function of ρ, we have,

(3)............$\rho = \pm\, r\sqrt{1 + \frac{n^2}{4}\sin^2 2\theta}$, the equation required.

The double sign shows that the curve consists of two equal branches in symmetrical position. The figure represents the curve, approximately, when $n = 4$.

[This solution gives the equations between the radius-vector and the *angle* subtended by the axis of x and the perpendicular upon the tangent. From it, however, we may readily obtain the equation between the radius-vector and the perpendicular (P) upon the tangent. We have $P = n\cos^2\theta - 1 = n(1 - \sin^2\theta) - 1 = n - 1 - n\sin^2\theta$; and from (3) we have $\sin\theta = \pm\sqrt{\frac{1}{2} \pm \frac{1}{2n}\sqrt{n^2 - \rho^2 + 4}}$.

$$\therefore\; P = \frac{n-2}{2} \mp \frac{1}{2}\sqrt{n^2 - 4\rho^2 + 4}\;\ldots\ldots(4).$$

Or, because $\rho : P :: \sin\theta : \sin\psi$ we have $\rho\sin\psi = P\sin\theta$, or, by substitution,

$$\rho\sin\psi = \left(\frac{n-2}{2} \mp \frac{1}{2}\sqrt{n^2 - 4\rho^2 + 4}\right)\sqrt{\frac{1}{2} \pm \frac{1}{2n}\sqrt{n^2 - \rho^2 + 4}}\;\ldots\ldots(5),$$

which corresponds with the eq. obtained by Prof. Evans by a different process of reasoning.—Ed.]

Dr. Jas. Matteson, of De Kalb Center, Ill., has sent us an elaborate solution of No. 5, which we are sorry our space will not permit us to publish.

Putting x, y and z for the three sides of the triangle, and 5, 7 and 9 for the three bisecting lines, and, moreover, putting $mx = y$ and $nx = z$, the Dr. gets, $m = .5814480988564$, and $n = 7991785252715$.

$$x = \frac{5}{\sqrt{mn - \dfrac{mn}{m+n}}},\ y = \frac{7}{\sqrt{m - \dfrac{mn^2}{(m+1)^2}}} \text{ and } z = \frac{9}{\sqrt{n - \dfrac{m^2n}{(n+1)^2}}}.$$

PROBLEMS.

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37. By G. Shaw, Kemble, Ontario, Canada.—Divide unity into three parts such that if each part be increased by unity the sums shall be three rational cubes.

38. By Asher B. Evans, A. M., Lockport, N. Y.—Prove that when x is infinite

$$\frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}} - \ldots\ldots\ldots\ldots \frac{(-1)^x}{\underline{|x}} = \frac{1}{e};$$

where e is the base of Napierian logarithms.

39. By Prof. C. M. Woodward, St. Louis, Mo.—Two weights are connected by a fine string which passes over a pulley; if the weights be 50 and 72 lbs., determine what stationary weight the string must be able to support, that it may just escape breaking during the motion.

40. By the Editor.—P and Q are two points, distant apart a miles on the bank of a straight canal, through which there flows, from Q towards P, a uniform current with the velocity of m miles per hour. R is a point on the opposite bank of the canal, at right angles from P with the line PQ. Two men, A and B, start at the same moment from the two points P and R; A starts from P and *walks* directly toward Q with the velocity of n miles per hour, and B starts from R, with a boat, and *rows*, with a constant effort that in the absence of a current, would carry him r miles per hour, and endeavors to join A by rowing continually directly towards him, and succeeds in joining A at the moment he arrives at Q. What is the width of the canal? or, distance between P and R?